Problem Set 6
Statistical Mechanics
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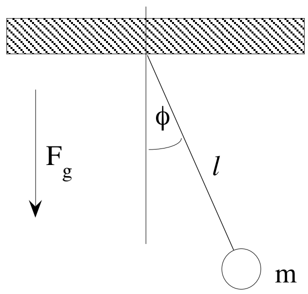
Problem 15: Thermal energy of a plane pendulum

(a) Calculate the mean thermal energy $\langle E \rangle$ of a plane pendulum (sketch) in the limiting case of small deflection ($\cos\phi \approx 1 - \phi^2/2$) quantum mechanically - as a function of the magnitude of the acceleration due to gravity g and the length of the pendulum l. Note: Consider the 1D harmonic oscillator and its eigenvalues $E_\nu = \hbar\omega(\nu + 1/2)$. How does ω relate to g and m?

(6 points)



(b) Sketch $\langle E \rangle/(\hbar\omega)$ and the dimensionless heat capacity C_V/k_B versus T/T_s, where T_s is a characteristic temperature of the pendulum. Indicate approximately in what temperature range the pendulum behaves classically. How long must the pendulum be so that you may calculate classically at $T = 300K$?

(6 points)

Problem 16: Defects in a solid

A crystalline solid contains N stationary but independent defects. Each defect possesses 5 possible energy states $\epsilon_1 = \epsilon_2 = 0$ and $\epsilon_3 = \epsilon_4 = \epsilon_5 = \Delta$.

(a) Find the partition function of the defects and the contribution of the defects to the entropy of the crystal as function of Δ and temperature T.

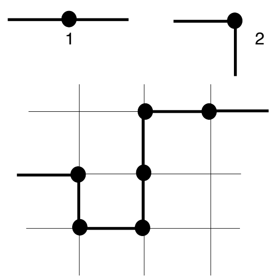
(6 points)

(b) What is the contribution of the defects to the internal energy of the solid in the limit $k_B T \gg \Delta$?

(3 points)

Problem 17: Polymer chain on a square lattice

The following sketch shows a path, i.e. a model polymer, on a square lattice (2D) composed of the two elements 1 and 2. Element 1 has the energy $\epsilon_1 = 0$ and element 2 has the energy $\epsilon_2 = \epsilon > 0$. That is straight segments are energetically more favorable than kinks. There are no interactions between the segments.



(a) Calculate the partition function for a polymer of length n (segments). Note that there are two kinks (right and left) of type 2.

(6 points)

(b) Calculate the internal energy of the polymer chain.

(3 points)

(c) Calculate the entropy of the polymer chain in the limit $T \to \infty$.

(3 points)