**Problem Set 5**
Statistical Mechanics
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**Problem 12:** Inversion temperature from the universal van der Waals equation

In class we have discussed the universal van der Waals equation

$$p = \frac{8t}{3v-1} - \frac{3}{v^2} \ .$$

(a) Obtain (numerically) the line $\mu_{JT} = 0$ in the $p$-$t$-plane (for $p > 0$ and $t > 0$). Here $\mu_{JT}$ is the Joule-Thomson coefficient.

(6 points)

(b) Look up the critical parameters for Argon, i.e critical temperature $T_c$, critical pressure $P_c$ and critical volume $V_c$. Use these values to obtain the 'van der Waals inversion temperature' corresponding to the inversion temperature in problem 8.

(3 points)

**Problem 13:** Phase separation in a binary mixture

In lecture 8 we had discussed the enthalpy of mixing, which for an ideal binary system consisting of components A and B is given by

$$\Delta_M G = nRT\left(x_A \ln x_A + x_B \ln x_B\right) \ .$$

Here $x_A + x_B = 1$. To this we add a simple interaction term, i.e.

$$\Delta G = nRT\left(x_A \ln x_A + x_B \ln x_B + \chi x_A x_B\right) \ .$$

Based on $\Delta G$ obtain the phase diagram of the AB-mixture in the $x_A$-$\chi$-plane, i.e. calculate the (binodal) line in the $x_A$-$\chi$-plane separating the area in which the mixture is homogeneous from the area in which the mixture 'breaks up' into A-rich and A-poor regions (analogous to the liquid-like and gas-like regions found in a system where the vdW pressure must be replaced by a constant). See also section 4.3 in Thermodynamics. Also calculate the (spinodal) line marking the stability limit $\partial^2 \Delta G / \partial x_A^2|_{T,P} = 0$. Include this line in your phase diagram. Can you say what this equation has to do with stability? Does it matter that you use $\Delta_M G$ instead of the full $G$ (please explain)?

(6 points)

**Problem 14:** Model magnet

A simple model for the thermal behavior of a magnet possess the energy eigenvalues $E_\nu = -Jm_\nu \langle m \rangle$, where $J > 0$ is a constant. In addition $\nu = 1, 2$ and $m_1 = 1$, $m_2 = -1$. Derive an implcit formula for the average magnetization $\langle m \rangle$ as function of the dimensionless temperature $T/T_c$ ($T_c = J/k_B$). Expect more than one solution for $T < T_c$. Which of these is/are the thermodynamic stable solution/s? Sketch the magnetization for $0 < T < \infty$.

(9 points)