Problem Set 1
Statistical Mechanics
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Problem 1: Surface and volume work

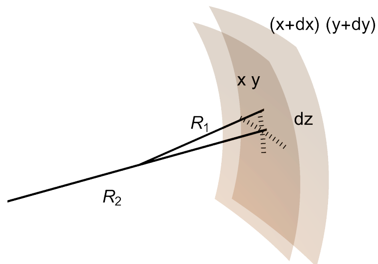
(a) What is the minimum work, in units of Joule, required to form the surface associated with a water mist having a particle radius of 5 micron? Assume that 1 liter of water at 50°C is to be turned into mist at 50°C. The surface tension of water at this temperature is 63 dyne/cm.

(3 points)

(b) Consider a liquid drop suspended from a nozzle. The shape of the drop is determined by the local pressure difference in the water versus the surrounding air $\Delta P(h)$, where the h-axis is the vertical symmetry axis of the drop, and the surface tension of water γ. Assuming the equilibrium condition $dE = -\Delta P dV + \gamma dA = 0$ show that the following equation holds

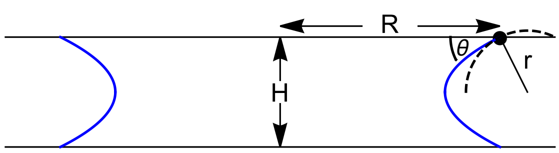
$$\Delta P(h) = \gamma(1/R_1 + 1/R_2) \ .$$

The quantities R_1 and R_2 are the principal radii of curvature of the drops surface at height h. Hint: Apply the above equilibrium condition to the following sketch. Use $dV = Adz$ and express dA in terms of R_1 and R_2.



(9 points)

(c) A liquid drop is confined between two parallel glass plates. The following sketch shows a vertical cut through the center of the drop.



Apply the equation from part (b) to calculate the force of attraction between the plates due to the liquid between them (neglect gravitation and use $R \gg r$). Note that the sign of the radii of curvature are different in this case, since r is outside the drop. Hint: The solution looks like this: $F = -....\gamma \cos\theta / H$. You should fill in the dots. Calculate the force explicitly for $\gamma = 0.0728$ N/m (water), $\theta \approx 0$, $R = 1$ cm and $H = 5$ μm.

(5 points)